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MATH 3303-01 : *Linear Algebra*
Due: Friday, April 15, 2005
(#29), (388-389), 1, 4, 9, 11, 14, 16
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In exercises 1 through 4, find a least squares solution to $A\mathbf{x} = \mathbf{b}$.

388-1

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \\ 0 & -1 \\ -1 & 1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \end{bmatrix}$$

$rref(A)$ shows that rank $A = 2$. The normal system $A^T A\mathbf{x} = A^T\mathbf{b}$ is

$$\begin{bmatrix} 6 & 1 \\ 1 & 3 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$$

Therefore,

$$\hat{\mathbf{x}} = \begin{bmatrix} \frac{24}{17} \\ -\frac{8}{17} \end{bmatrix}$$

388-4

$$A = \begin{bmatrix} -1 & 0 & 0 & 4 \\ 4 & -2 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 2 & -2 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} -1 \\ 1 \\ 2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$rref(A)$ shows that rank $A = 4$. The normal system $A^T A\mathbf{x} = A^T\mathbf{b}$ is

$$\begin{bmatrix} 30 & -8 & 3 & -2 \\ -8 & 8 & -4 & 0 \\ 3 & -4 & 6 & -2 \\ -2 & 0 & -2 & 21 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 13 \\ -2 \\ 2 \\ -3 \end{bmatrix}$$

Therefore, $\hat{\mathbf{x}} = \begin{bmatrix} \frac{25}{49} \\ \frac{85}{196} \\ \frac{17}{49} \\ -\frac{3}{49} \end{bmatrix}$

In exercises 7 through 10, find a least squares line for the given data points.

388-9 (2,3), (3,4), (4,3), (5,4), (6,3), (7, 4)

When you put those data points into matrices A and $\mathbf{b}$, you get

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 1 \\ 4 & 1 \\ 5 & 1 \\ 6 & 1 \\ 7 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 3 \\ 4 \\ 3 \\ 4 \\ 3 \\ 4 \end{bmatrix}$$

Since $\hat{\mathbf{x}} = (A^T A)^{-1} A^T \mathbf{b}, \hat{\mathbf{x}} = \begin{bmatrix} \frac{3}{35} \\ \frac{109}{35} \end{bmatrix}$. Since (in this case) $\hat{\mathbf{x}} = \begin{bmatrix} m \\ b \end{bmatrix}, y = \frac{3}{35}x + \frac{109}{35}$.

In Exercises 11 and 12, find a quadratic least squares polynomial for the given data.

388-11 (0, 3.2), (0.5, 1.6), (1, 2), (2, -0.4,) (2.5, -0.8), (3, -1.6), (4, 0.3), (5, 2.2)

When you put those data points into matrices A and $\mathbf{b}$, you get

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0.25 & 0.5 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \\ 6.25 & 2.5 & 1 \\ 9 & 3 & 1 \\ 16 & 4 & 1 \\ 25 & 5 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 3.2 \\ 1.6 \\ 2 \\ -0.4 \\ -0.8 \\ -1.6 \\ 0.3 \\ 2.2 \end{bmatrix}.$$

$$\hat{\mathbf{x}} = (A^T A)^{-1} A^T \mathbf{b}, \text{ so } \hat{\mathbf{x}} \approx \begin{bmatrix} .5718 \\ -3.1314 \\ 3.4627 \end{bmatrix}$$

Thus, $y \approx .5718x^2 - 3.1314x + 3.4627$.

388-14 A steel producer gathers the following data.

Year	1997	1998	1999	2000	2001	2002
Annual sales (millions of dollars)	1.2	2.3	3.2	3.6	3.8	5.1

Represent the years 1997,...,2002 as 0, 1, 2, 3, 4, 5, respectively, and let x denote the year. Let y denote the annual sales (in millions of dollars).

(a) Find the least squares line relating x and y.

When you put those data points into matrices A and $\mathbf{b}$, you get

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \\ 5 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 1.2 \\ 2.3 \\ 3.2 \\ 3.6 \\ 3.8 \\ 5.1 \end{bmatrix}$$

$\hat{\mathbf{x}} = (A^T A)^{-1} A^T \mathbf{b}$, so $\hat{\mathbf{x}} \approx \begin{bmatrix} .6971 \\ 1.4571 \end{bmatrix}$.

Thus, $y \approx .6971x + 1.4571$.

(b) Use the equation obtained in (a) to estimate the annual sales for the year 2006.

$y \approx .6971(9) + 1.4571 \rightarrow y \approx 7.7314$.

389-16 The distributor of a new car has obtained the following data.

Number of Weeks After Introduction of Car	*Gross Receipts per Week* (millions of dollars)
1	0.8
2	0.5
3	3.2
4	4.3
5	4
6	5.1
7	4.3
8	3.8
9	1.2
10	0.8

Let x denote the gross receipts per week (in millions of dollars) t weeks after the introduction of the car.

(a) Find a least squares quadratic polynomial for the given data.

When you put those data points into matrices A and $\mathbf{b}$, you get

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \\ 16 & 4 & 1 \\ 25 & 5 & 1 \\ 36 & 6 & 1 \\ 49 & 7 & 1 \\ 64 & 8 & 1 \\ 81 & 9 & 1 \\ 100 & 10 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 0.8 \\ 0.5 \\ 3.2 \\ 4.3 \\ 4 \\ 5.1 \\ 4.3 \\ 3.8 \\ 1.2 \\ 0.8 \end{bmatrix}$$

$$\hat{\mathbf{x}} = (A^TA)^{-1}A^T\mathbf{b}, \text{ so } \hat{\mathbf{x}} \approx \begin{bmatrix} -0.2129 \\ 2.3962 \\ -2.1833 \end{bmatrix}.$$

Thus, $y \approx -0.2129x^2 + 2.3962x - 2.1833$.

(b) Use the equation in part (a) to estimate the gross receipts 12 weeks after the introduction of the car.

$y \approx -0.2129(12)^2 + 2.3962(12) - 2.1833 \rightarrow y \approx -4.0833$.

This is, of course, impossible, but it goes to show you what extrapolation can do to you ...